ENDPOINT ESTIMATES FOR BILINEAR SPHERICAL MAXIMAL FUNCTION

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Abstract:

The study of maximal averaging operators plays a fundamental role in analysis. One such operator is the spherical maximal function, defned as

$$A_*f(x) = \sup_{t>0} \Big| \int_{\mathbb{S}^{d-1}} f(x-ty)d\sigma(y)\Big|.$$

The L^p-boundedness of A_* was proved by Stein (1976) for $p > \frac{d}{d-1}, d \geq 3$ and Bourgain (1986) in dimension $d = 2$ for $p > 2$.

In this talk, we consider the bilinear analogue of the spherical maximal function given by

$$M(f,g)(x) = \sup_{t>0} \Big| \int_{\mathbb{S}^{2d-1}} f(x-ty)g(x-tz)d\sigma(y,z)\Big|.$$

In dimension $d \geq 2$, Lee and Jeong (2020) proved sharp $L^p(\mathbb{R}^d) \times L^q(\mathbb{R}^d) \to L^r(\mathbb{R}^d)$ bounds for bilinear spherical maximal function when $r > \frac{d}{2d-1}$. For $r = \frac{d}{2d-1}$, they obtained restricted weak type bounds in dimension $d \geq 3$. Dosidis and Ramos (2022) proved sharp bounds in dimension $d = 1$ for $p, q > 2$ and $\frac{1}{r} = \frac{1}{p} + \frac{1}{q}$. We prove restricted weak type estimate at the endpoints for bilinear spherical maximal function in dimensions $d = 1, 2$. We also prove sharp L^p improving bounds for localised bilinear spherical maximal function.

This is a joint work with Ankit Bhojak, Saurabh Shrivastava and Kalachand Shuin.